

Experimental validation of FE models using FE-model-based scanning laser Doppler vibrometry

T. Burmeister ¹, D. Mastrodicasa ², B. Arnoldi ¹, T. Schröder ¹, R. Araújo ², E. Di Lorenzo ², D. Günther ²

¹ Optomet GmbH, Darmstadt, Germany

² Siemens Digital Industries Software, Leuven, Belgium

Abstract

The increasing use of lightweight and flexible aircraft structures in pursuit of higher efficiency requires reliable prediction of relevant structural dynamics, including flutter-critical behavior. Although aeroelastic behavior is primarily assessed numerically, its accuracy depends on experimentally validated structural finite element (FE) models. A validation gap arises because FE models with many degrees of freedom are typically compared to experimental modal analyses, limited by discrete sensor locations and therefore restricted to lower eigenmodes and frequency ranges. This paper presents a high-resolution experimental validation approach for the validation of structural FE models used in aerospace-relevant dynamic analyses. The focus lies on obtaining reliable structural dynamic data as a prerequisite for future investigations of flutter-critical behavior and modal coupling phenomena in the absence of aerodynamic loading. A simplified flat-plate structure, used as a laboratory analogue for bending-dominated aerospace components such as control surfaces is experimentally characterized using full-field vibration measurements obtained by scanning laser-Doppler vibrometry. Experimental modal analysis provides eigenfrequencies, damping coefficients, and mode shapes with high spatial resolution over an extended frequency range. The FE model is validated through modal correlation, including frequency comparison and modal assurance criterion (MAC) evaluation, providing a basis for subsequent model updating. The experimentally validated structural models provide the required foundation for future flutter-relevant investigations and broader aerospace dynamic analyses. The underlying objective is to enable dry wind tunnel testing of aerospace components, and potentially complete aircraft, which would reduce experimental complexity, cost, and thereby development risk.

1 Introduction

The increasing use of lightweight and flexible aircraft structures requires reliable prediction of structural dynamic behavior throughout the development process. This is particularly important for components whose dynamic response may contribute to aeroelastic instabilities. Aeroelasticity describes the interaction between aerodynamic forces, structural elasticity and inertia, and therefore combines aerodynamic, structural-dynamic and elastic effects. Among the resulting phenomena, flutter is one of the most critical, since it represents a self-excited dynamic instability that can lead to rapidly increasing vibration amplitudes and, ultimately, structural failure. Accurate flutter prediction therefore depends strongly on reliable structural dynamic models, including natural frequencies, damping values and mode shapes. Numerical aeroelastic analyses consequently require experimentally validated finite element (FE) models as a reliable structural basis [1], [2], [3].

Dry wind tunnel testing concepts offer a potential route to reduce the complexity and cost of aeroelastic investigations by studying structural dynamic behavior under controlled laboratory conditions [4]. Instead of generating aerodynamic loads through physical airflow, such approaches can represent aeroelastic effects by externally applied actuator forces, equivalent aerodynamic loads, or frequency response function (FRF)-

based aeroelastic formulations [4], [5]. Their applicability therefore depends on the availability of reliable structural dynamic information, either in the form of validated structural models or experimentally measured structural response data with sufficient modal accuracy. The present study addresses this prerequisite by focusing on the experimental validation of a structural FE model rather than on the reproduction of flutter itself.

Experimental modal analysis is commonly used for structural model validation [6]. Conventional approaches based on pointwise sensors, such as accelerometers, provide accurate local response information but are limited when applied to lightweight or spatially complex structures. Measurement density is typically constrained by sensor availability, installation effort and cabling, while attached sensors may locally affect the dynamic behavior due to mass loading [7]. Consequently, experimentally identified mode shapes are often spatially sparse and may require interpolation before comparison with FE results. This can limit modal correlation, especially for higher-order modes, local deformation patterns and modal coupling phenomena.

Scanning laser Doppler vibrometry (SLDV) provides a non-contact alternative for high-resolution vibration measurements without adding mass to the test object [8]. For FE model validation, however, high spatial resolution alone is not sufficient. A practical challenge remains in establishing spatial consistency between experimental measurement locations and the numerical model. Such consistency can reduce mapping effort, avoid unnecessary interpolation and enable more reliable experimental-numerical modal correlation.

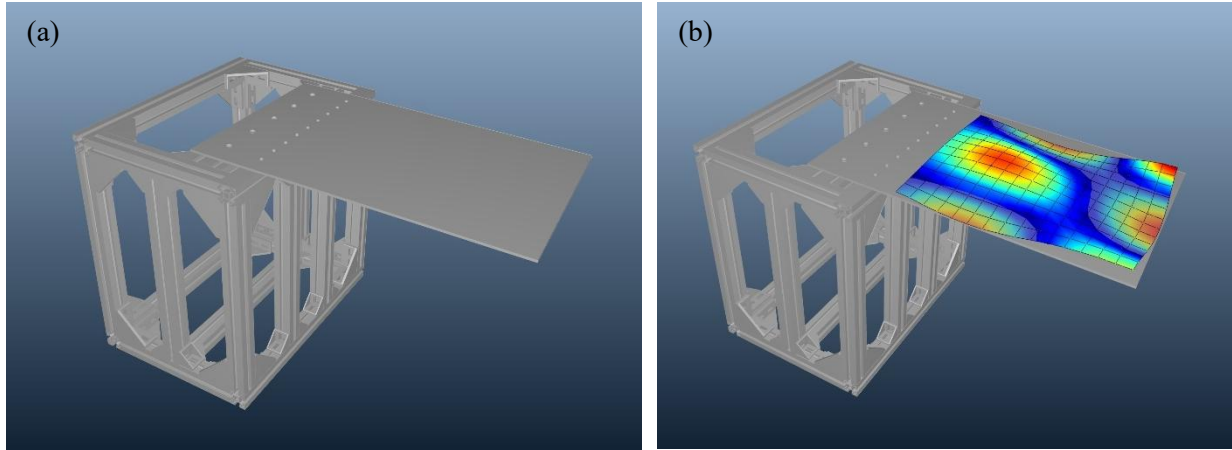


Figure 1: Measurement and analysis environment SMART Lab. (a) Imported FE model. (b) Operational deflection shape visualized directly on the FE model.

This paper presents an FE-based experimental validation workflow using high-resolution SLDV, demonstrated through a measurement campaign conducted at Siemens Digital Industries Software in Leuven. A flat-plate structure used as a simplified laboratory analogue for bending-dominated aerospace components such as control surfaces, is clamped at one end and excited by a shaker under controlled laboratory conditions. The FE model is imported into the measurement environment **Figure 1(a)**, and measurement points are directly placed on its surface. Experimental modal parameters and mode shapes are identified and compared with numerical results using frequency correlation and the modal assurance criterion (MAC). **Figure 1(b)** shows an exemplary operational deflection shape (ODS) directly visualized on the FE model.

The main contribution of this work is the demonstration of an FE-model-based SLDV workflow for direct experimental-numerical modal correlation. By combining geometry-based scan definition, non-contact vibration measurements, modal analysis and standardized validation data export, the workflow provides a consistent basis for subsequent model updating and future flutter-relevant investigations.

2 FE-based experimental validation workflow

The experimental validation workflow is implemented in the Optomet SMART Lab software as an integrated measurement and analysis environment. The structural FE model is first imported into the software and used as the spatial reference for the measurement definition. A boundary shape is created using the FE model's surface as highlighted by the red outline in **Figure 2(a)**. A grid of equidistant measurement points can then be generated inside the boundary shape. While this is a simple task for the present flat plate, equidistant grids of measurement points can also be generated on complex, curved surfaces to ensure consistent results across setups.

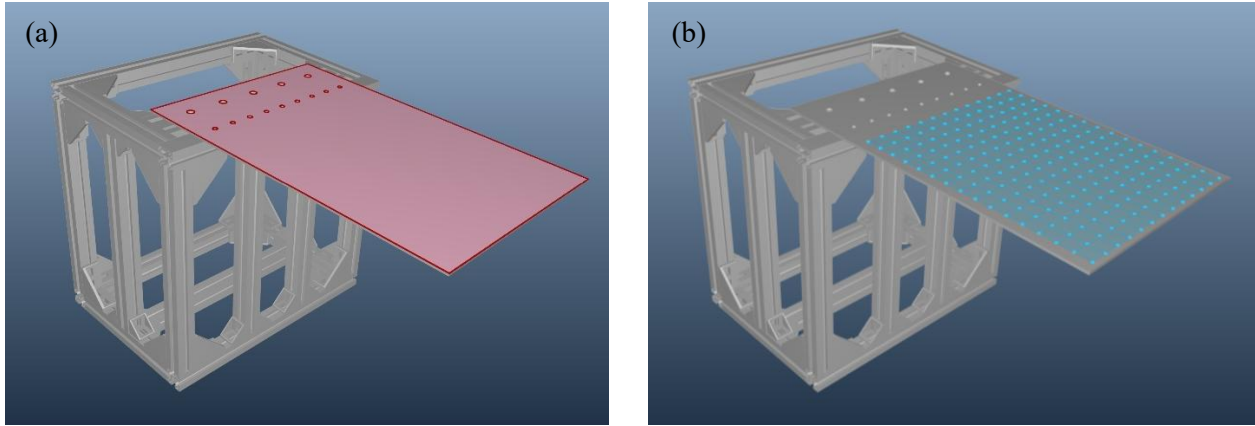


Figure 2: Measurement point creation in SMART Lab: (a) Generation of a boundary shape (red outline) based on the FE model's geometry. (b) Populating the boundary with an equidistant grid of measurement points.

Spatial consistency between the physical test object and the FE model is established in SMART Lab through a two-step calibration workflow. First, the imported FE geometry is aligned with the image captured by the integrated 4K camera of the scanning laser Doppler vibrometer, hereafter referred to as the SLDV system. This algorithm-supported process enables calibration markers to be placed consistently in both the camera image and the 3D model.

In the second step, automatic laser calibration uses these reference points to determine the precise relative position of the test object and the vibrometer's laser beam. As a result, SMART Lab creates a spatially consistent virtual representation of the complete measurement setup in 3D space — a digital twin that links the physical object, FE model, camera view, and laser position.

After calibration, the SLDV system measures the vibration response at the previously defined measurement points, while simultaneously recording the data of the reference sensors.

Frequency response functions (FRFs) are calculated for each measurement point. At a user-defined frequency, the ODSs can be visualized directly on the imported FE model by extracting amplitude and phase information from the FRF data.

Experimental modal analysis can then be performed on the FRF data to identify natural frequencies, damping ratios, and mode shapes. This analysis can be carried out either directly in SMART Lab or in external modal analysis software. The complete dataset, including all pointwise FRFs and the associated geometry, can be exported in UFF, ATFX, MAT, and HDF5 formats.

In this work, further processing and comparison of numerical and experimental data is performed in Simcenter 3D.

3 Experimental setup and data acquisition

The experimental setup used for the validation measurements is shown in **Figure 3**. The test object consists of a flat plate, which is rigidly clamped along one of its four edges. This boundary condition represents a simplified cantilever-like configuration and provides a controlled structural test case for the investigation of bending-dominated vibration modes.

The vibration response of the plate was measured using an Optomet SMART series SLDV system positioned in front of the test object and directed toward the plate's surface. The SLDV system enables non-contact measurement of the out-of-plane vibration response at multiple predefined measurement locations on the structure. In addition to the optical measurement system, the plate is equipped with accelerometers to provide pointwise reference measurements. These reference sensor signals are acquired simultaneously by the SLDV system, ensuring synchronized recording of the optically measured vibration data and the corresponding reference measurements.

The structure is excited by a uniaxial shaker. The excitation force is introduced at a defined location on the plate, and a force sensor is installed at the excitation point to measure the dynamic force transmitted from the shaker into the structure, thereby decoupling the shaker from the test structure. The excitation signal for the shaker is generated directly by the SLDV system, allowing the excitation and response measurements to be integrated within the same measurement workflow.

This setup enables a controlled experimental modal test in which the applied excitation force and the resulting structural response are acquired simultaneously. The measured force and response data form the basis for the subsequent calculation of frequency response functions and the identification of modal parameters.

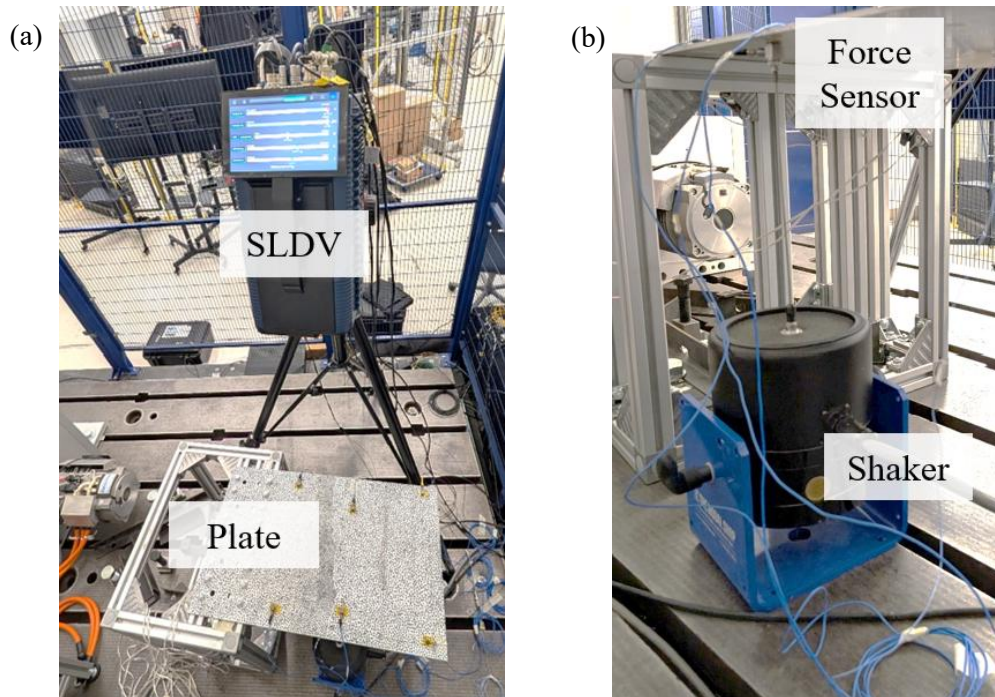


Figure 3: Experimental setup consisting of the SLDV system, clamped flat plate, uniaxial shaker, force sensor and accelerometer. (a) Top view. (b) Side view.

3.1 Measurement procedure and acquisition parameters

The measurement was carried out using a chirp excitation signal generated by the SLDV system. The chirp ranges from 5 Hz to 500 Hz with a signal duration of 2 s. The signal generator sample rate was set to 14 kHz.

The vibration response was acquired by the SLDV system using a velocity measurement range of 50 mm/s and a measurement frequency range of 2 kHz. All data is sampled with a frequency of 12 kHz. For each scan point, four blocks were recorded, each with a duration of 2 s. A delay of 1 s was applied between consecutive blocks to allow vibrations from previous chirps to fade.

The measurement grid was defined with a point spacing of 30 mm, resulting in 208 measurement points on the plate. At each point, the vibration response data (velocity and displacement) was acquired along with the corresponding reference channels for acceleration, the shaker's force input and the excitation signal. This data enables subsequent frequency response function calculation and modal parameter identification.

4 Experimental modal data evaluation

The measurement data was evaluated over the full excitation bandwidth. Owing to the non-contact nature of the SLDV system, the spatial response field could be measured over the complete scan grid without introducing additional mass or stiffness to the test object. This enabled the reconstruction of experimental mode shapes with high spatial resolution, without introducing additional sensor-induced mass loading or locally altering the dynamic behavior of the structure.

The acquired measurement data were evaluated to characterize the dynamic response of the clamped plate and to derive the experimental modal basis for the subsequent FE correlation. This chapter first presents the frequency-domain response obtained from the SLDV scan and uses selected operating deflection shapes to illustrate the measured spatial vibration behavior. The experimentally identified mode shapes are then summarized and prepared for comparison with the numerical modal results.

4.1 FRF acquisition and ODS

Based on the synchronously acquired excitation force and out-of-plane velocity response, velocity-over-force FRFs were calculated for all measurement points of the SLDV scan grid. The averaged FRF magnitude over all scan points is shown in **Figure 4** and provides an overview of the dynamic response of the clamped plate within the investigated frequency range. The black vertical lines indicate the modal frequencies identified by the SLDV-based experimental modal analysis discussed in Section 4.2.

To qualitatively assess the spatial response behavior at selected resonances, operational deflection shapes were extracted from the complex FRF data at representative modal frequencies. **Figure 5** shows four selected ODSs at frequencies identified as modal frequencies in by experimental modal analysis as described in Section 4.2. The ODSs visualize the amplitude and phase distribution of the measured vibration response directly on the FE model and illustrate the increasing spatial complexity of the plate deformation at higher frequencies.

It should be noted that ODSs represent the operating response at a given excitation frequency and are therefore not identical to experimentally identified mode shapes. However, they provide an intuitive qualitative representation of the measured structural dynamics and support the subsequent interpretation of the experimental mode shapes presented in the following section.

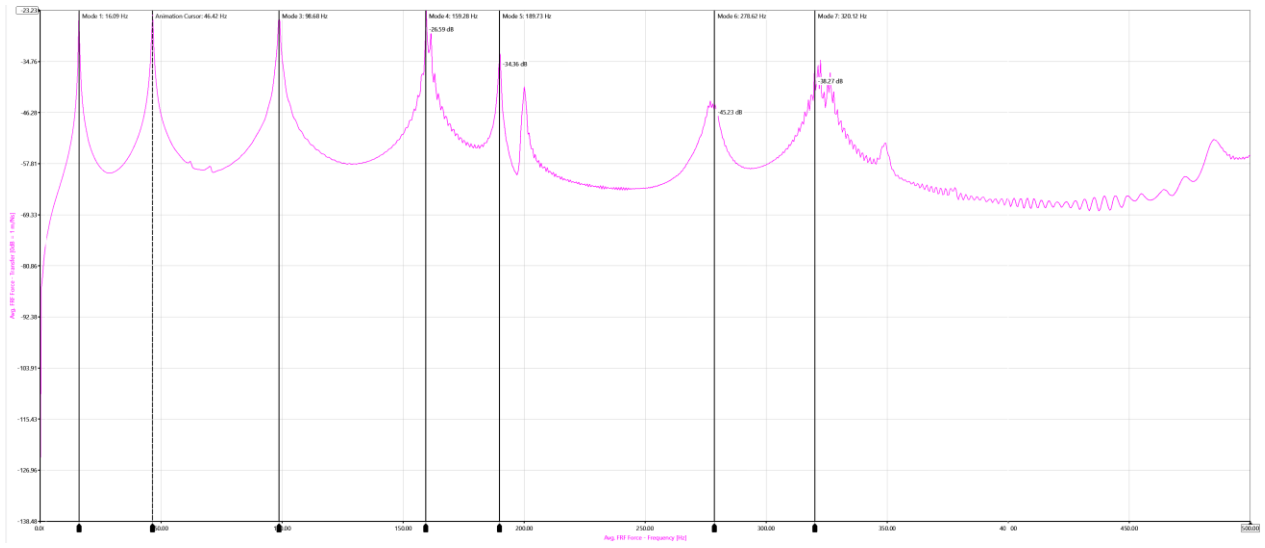
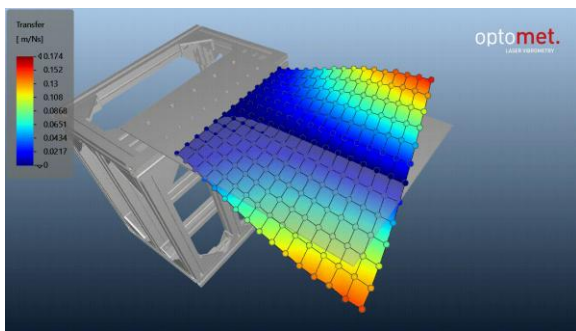
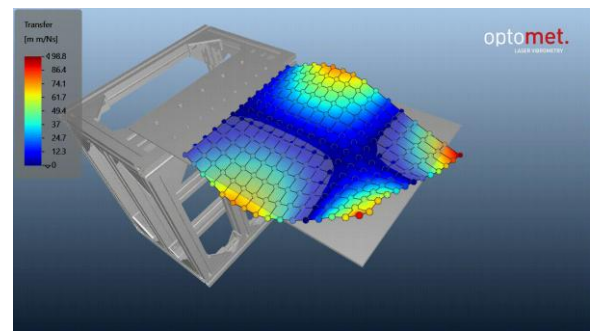


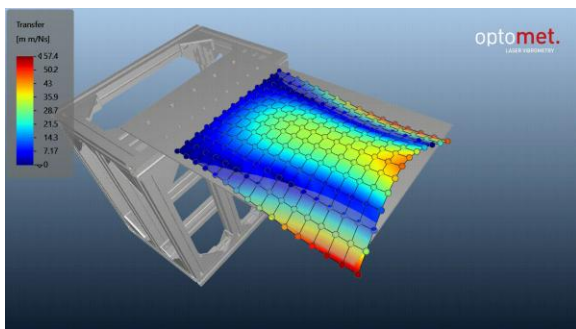
Figure 4: Velocity-over-force FRF magnitude averaged over all measurement points. Black vertical lines indicate the modal frequencies identified from the SLDV-based experimental modal analysis presented in Section 4.2.



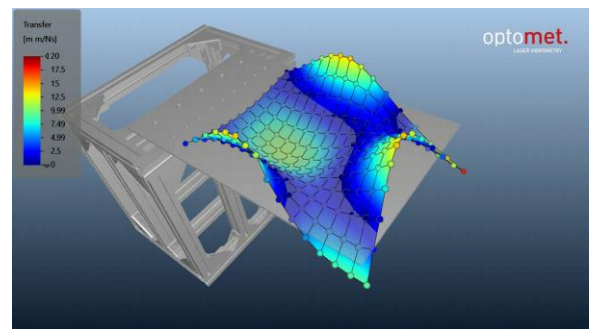
(a) 46.42 Hz



(b) 159.29 Hz



(c) 189.73 Hz



(d) 320.12 Hz

Figure 5: Selected ODSs extracted from velocity-over-force FRFs at modal frequencies identified in Section 4.2: (a) 46.42 Hz, (b) 159.29 Hz, (c) 189.73 Hz, and (d) 320.12 Hz.

4.2 Experimental mode shapes

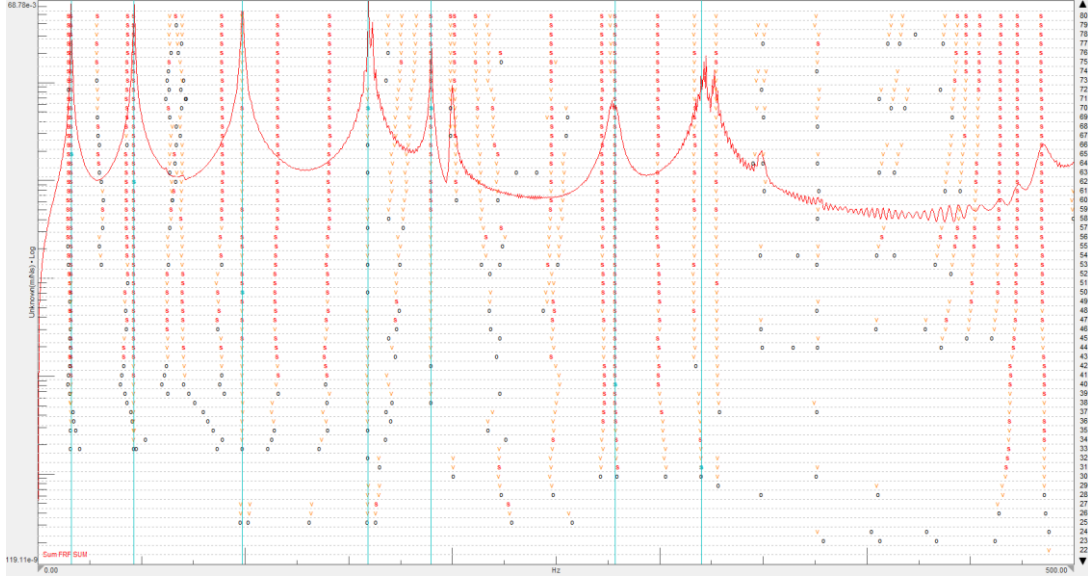


Figure 6: Stabilization diagram obtained from the experimental modal analysis of the clamped plate using velocity-over-force FRFs over the 5–500 Hz excitation range. Stable poles are used to support the selection of physical modes for subsequent experimental-numerical correlation.

Figure 6 shows the stabilization diagram obtained from the modal identification procedure. The diagram was used to assess the stability of the identified poles with respect to increasing model order and to distinguish physical structural modes from numerical or noise-related poles. The selection of modal parameters was based on the consistency of the identified poles in terms of frequency, damping, and mode-shape behavior.

The identified modal parameters of the selected modes, in terms of natural frequencies and modal damping, are summarized in **Table 1**.

Table 1: Selected experimental modal parameters

Mode #	Natural Frequency (Hz)	Modal Damping (%)
1	16.09	0.59
2	46.42	0.55
3	98.68	0.10
4	159.28	0.01
5	189.73	0.10
6	278.62	0.77
7	320.12	0.10

Selected experimental mode shapes are shown in **Figure 7**. Complex mode shapes are used for visualization because they retain the amplitude and phase information of the identified modal vectors. For the subsequent

correlation with the FE results, the experimental mode shapes were converted to real-valued mode shapes and scaled to unit modal mass.

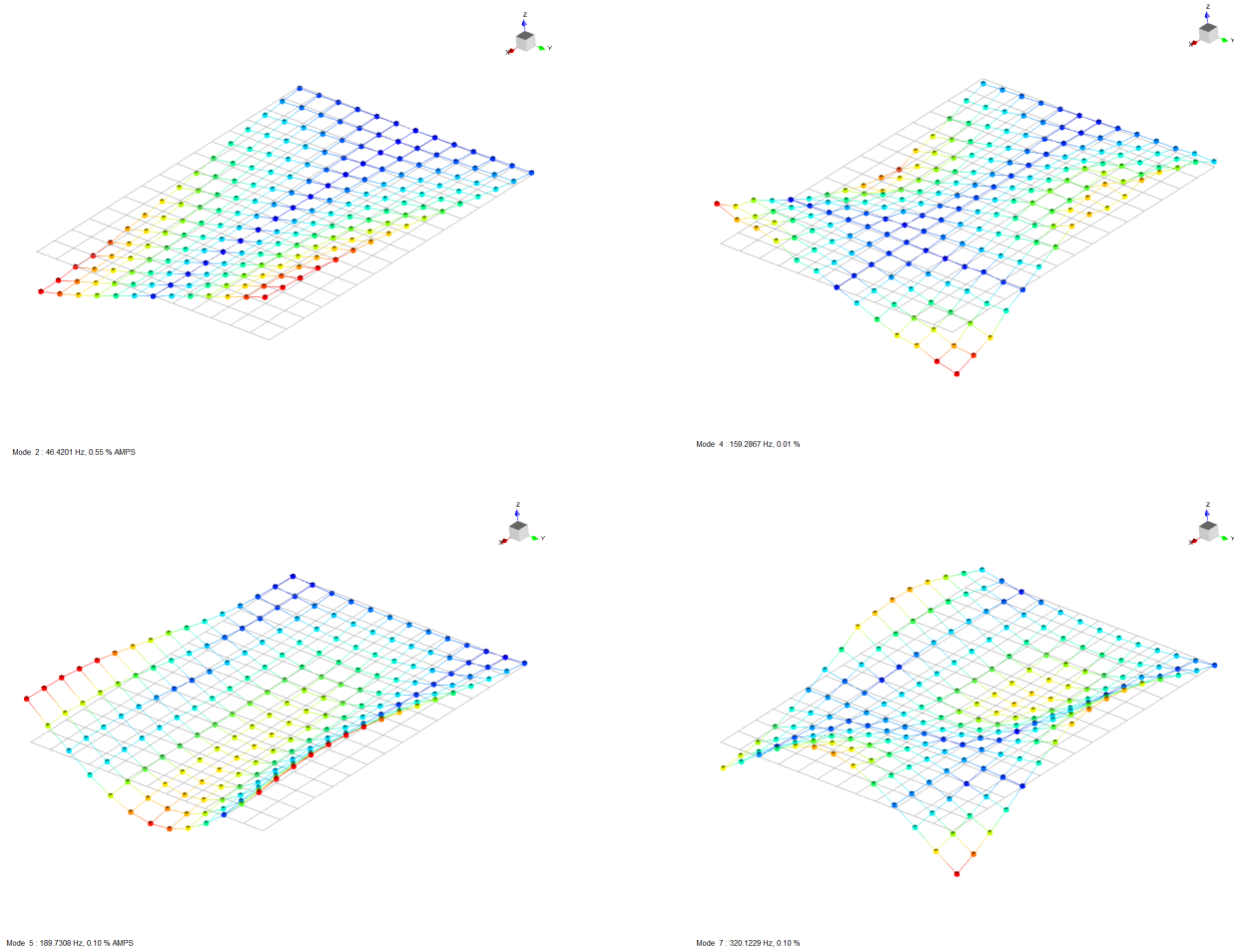


Figure 7: Selected complex experimental mode shapes identified from the SLDV-based modal analysis of the clamped plate: Mode 2 at 46.42 Hz, Mode 4 at 159.29 Hz, Mode 5 at 189.73 Hz, and Mode 7 at 320.12 Hz.

The selected experimental modes provide the experimental basis for the numerical comparison discussed in the following chapter. In particular, the high spatial density of the SLDV measurement enables a direct qualitative comparison with the corresponding finite element mode shapes and supports the MAC-based correlation between experiment and simulation.

5 FE model and experimental-numerical correlation

The experimentally identified modal parameters were compared with the results of a numerical modal analysis performed on a finite element model of the test setup. The numerical model includes both the plate, corresponding to the physical structure under test, and the supporting fixture. The interface between the plate and the fixture was modelled to represent the clamped boundary condition used in the experimental setup. This modelling approach allows the influence of the fixture and the boundary-condition representation to be included in the numerical modal analysis, rather than considering the plate as an isolated structure.

Figure 8 shows the finite element model used for the numerical modal analysis. Since the experimental measurement points were defined on the structural geometry with high spatial density, the measured mode shapes could be compared directly with the corresponding numerical mode shapes. This spatial consistency is particularly important for the evaluation of mode-shape correlation, as it reduces the need for additional interpolation between experimental and numerical data.

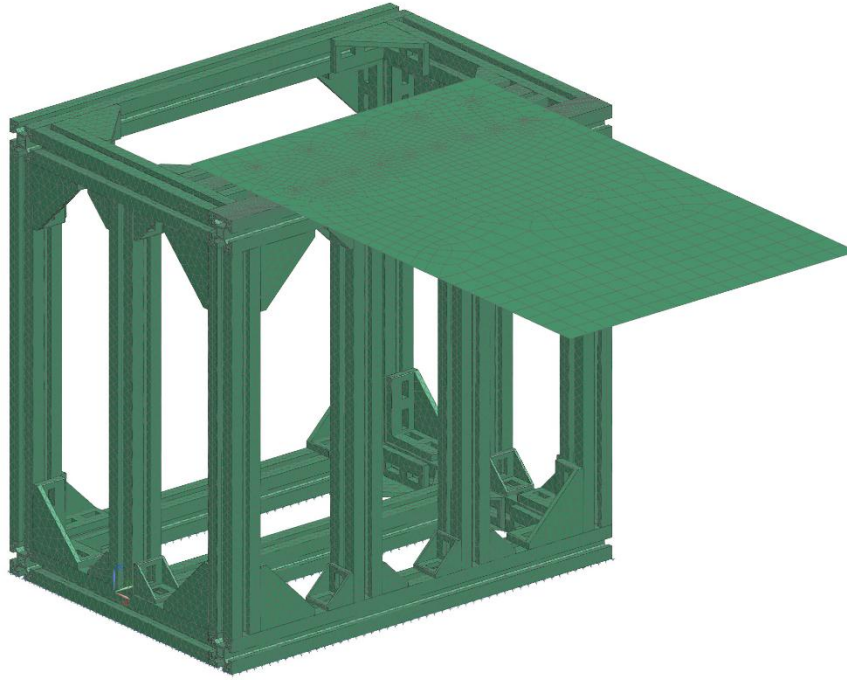


Figure 8: Finite element model of the clamped plate test setup, including both the plate and the supporting fixture. The interface between the plate and the fixture represents the clamped boundary condition applied in the experimental measurement.

Simcenter 3D was used to solve the eigenproblem and get natural frequencies and mode shapes. The frequency correlation is summarized in **Table 2**. The relative frequency deviation was calculated with respect to the experimental natural frequency according to:

$$\Delta f = \frac{f_{\text{num}} - f_{\text{exp}}}{f_{\text{exp}}} \cdot 100$$

where f_{exp} and f_{num} denote the experimental and numerical natural frequencies, respectively.

The results show good agreement between the experimental and numerical natural frequencies for the correlated mode pairs. The absolute frequency deviations remain below 1.6 %, indicating that the numerical model reproduces the global dynamic stiffness and inertia properties of the test setup with good accuracy in the investigated frequency range.

Table 2: Frequency correlation between experimental and numerical modes

Experimental mode	Exp. Natural Frequency (Hz)	Numerical mode	Num. Natural Frequency (Hz)	Δf (%)
1	16.09	1	16.08	-0.11
2	46.42	2	46.74	+0.69
3	98.68	3	98.83	+0.14
4	159.28	6	161.82	+1.59
5	189.73	8	186.98	-1.45
6	278.62	9	279.31	+0.25
7	320.12	10	321.40	+0.40

In addition to the comparison of natural frequencies, the spatial correlation between experimental and numerical mode shapes was evaluated using the modal assurance criterion. The resulting MAC matrix is shown in **Figure 9**. The MAC analysis was used to identify corresponding mode pairs and to assess the consistency of the measured and simulated deformation patterns. High MAC values on the selected mode pairs indicate good agreement between the experimental and numerical modal vectors, whereas lower off-diagonal values support the separation of the individual modes.

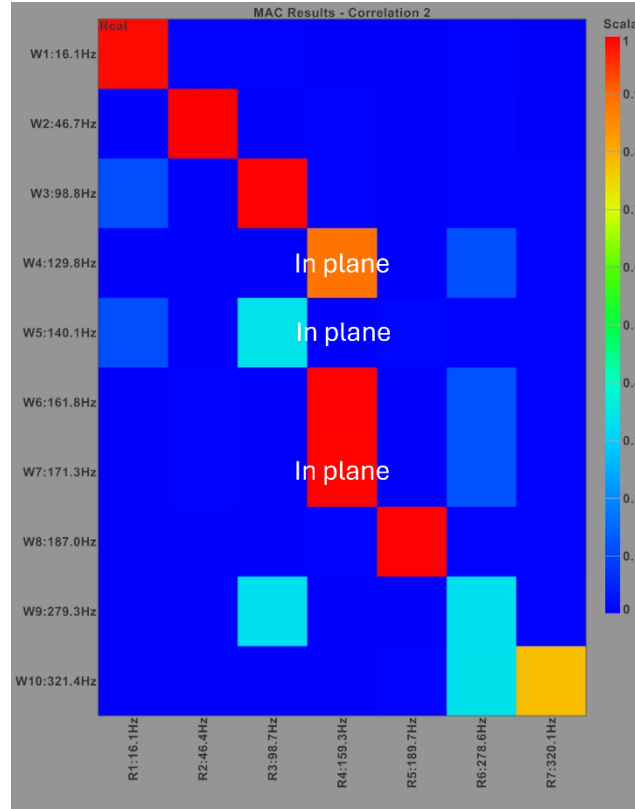


Figure 9: MAC matrix comparing the experimentally identified mode shapes with the numerical mode shapes from the finite element model. The matrix is used to identify corresponding mode pairs and to assess the spatial consistency between experimental and numerical modal vectors.

The selected experimental and numerical mode shapes are compared qualitatively in **Figure 10**. The comparison shows that the main deformation characteristics of the selected experimental modes are consistently reproduced by the corresponding numerical modes. This agreement supports the validity of the numerical model for representing the structural dynamics of the clamped plate within the investigated frequency range. Although the frequency agreement is generally good, some discrepancies remain in the mode-shape comparison and in the MAC results. These differences may be attributed to uncertainties in the representation of the clamped boundary condition, small deviations between the numerical geometry and the physical test setup, and the influence of the fixture on the measured structural response. Furthermore, the SLDV measurement captures the out-of-plane vibration response on the observed side of the structure, whereas the finite element model describes the full three-dimensional displacement field. Consequently, modes with pronounced in-plane components or local three-dimensional deformation patterns may show reduced correlation with the measured data.

Overall, the comparison demonstrates that the FE model captures the relevant dynamic behavior of the clamped plate with good accuracy for the correlated modes. The validated model therefore provides a suitable structural basis for subsequent investigations, including model updating and future flutter-relevant dynamic studies.

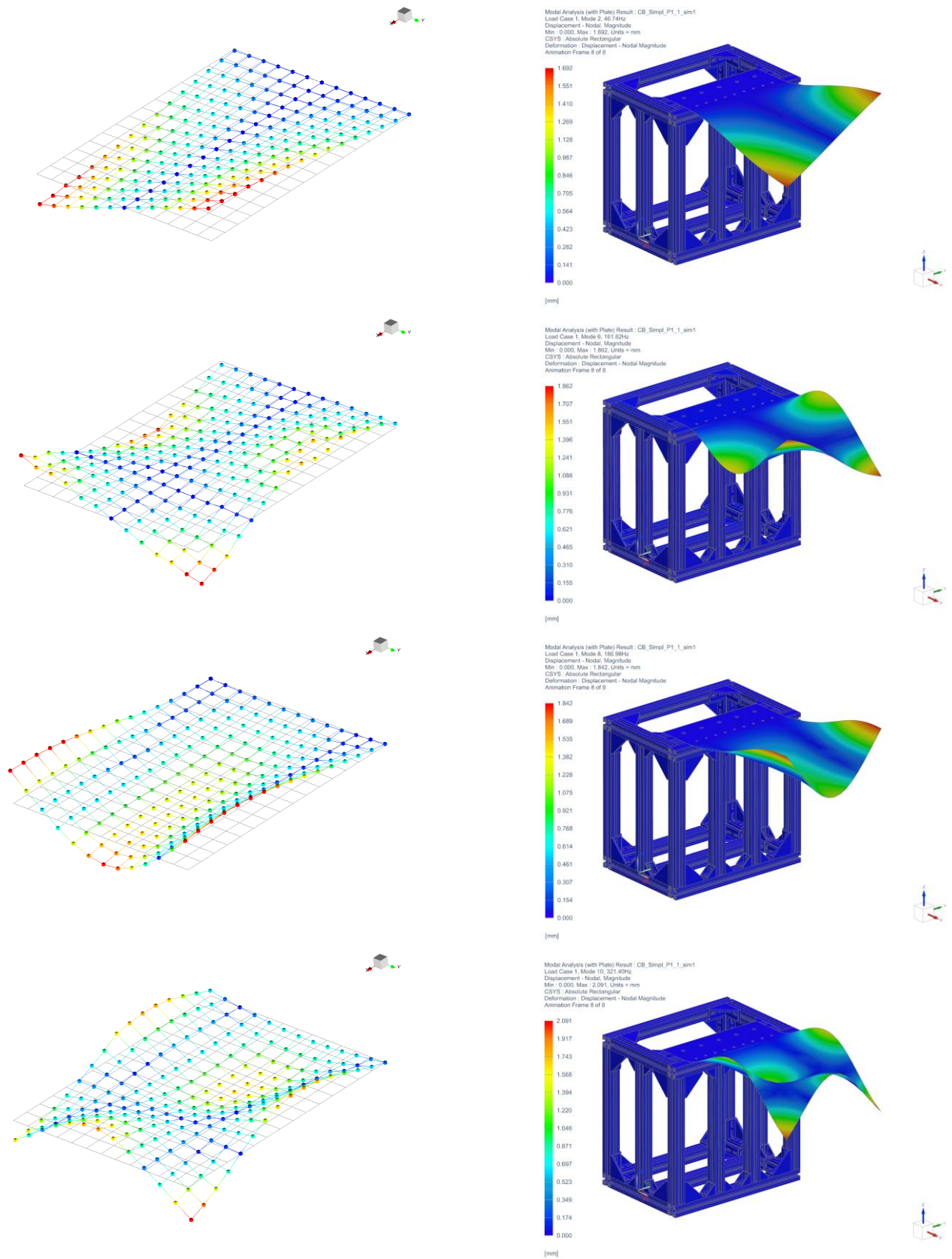


Figure 10: Qualitative comparison of selected experimental and numerical mode shapes. The experimental complex modes 2, 4, 5, and 7 are compared with the numerical modes 2, 6, 8, and 10, respectively.

6 Discussion

The correlation between experimental and numerical analysis presented in the previous chapter shows that the FE-model-based SLDV workflow provides a suitable basis for structural model validation. Rather than focusing only on the agreement of individual modal parameters, the main methodological value lies in the consistent link between scan definition, spatial calibration, vibration measurement and modal correlation. By defining the measurement grid directly on the imported structural geometry, the workflow reduces manual mapping effort and supports a more direct comparison between measured and numerical mode-shape data. This becomes increasingly relevant for higher-order modes and more complex structures, where sparse measurement grids or inconsistent spatial mapping can introduce additional uncertainty.

A relevant practical aspect is the distinction between the geometry used for measurement definition and the mesh used for numerical simulation. Smart Lab can use surface geometry data for scan definition, whereas FE simulation software requires a structural model with appropriate element formulation, material properties and boundary conditions. The surface representation used for optical measurement planning is therefore not necessarily identical to the numerical mesh used for modal analysis. Since mesh topology and modelling assumptions influence the simulated modal results, this distinction should be considered when interpreting experimental-numerical correlation results.

The present measurements were performed with a single SLDV measurement direction and therefore primarily capture the out-of-plane vibration response on the observed side of the structure. For the investigated flat plate, this is sufficient to characterize the dominant bending behavior. However, the FE model describes the full three-dimensional displacement field and may include modes with pronounced in-plane components. Such modes can show reduced correlation if only out-of-plane measurement data are available. Future work using a 3D scanning vibrometer system could improve the correlation of modes with significant in-plane or local three-dimensional deformation components.

With respect to flutter-related structural validation, the presented workflow should be understood as a preparatory step rather than a reproduction of aeroelastic behavior itself. Dry wind tunnel and ground flutter simulation approaches aim to investigate flutter-relevant behavior without generating aerodynamic loads through physical airflow. Instead, aerodynamic effects are represented by externally applied actuator forces, equivalent or condensed aerodynamic loads, or FRF-based aeroelastic formulations that combine experimentally measured structural response data with calculated aerodynamic representations [1], [9]. In such approaches, reliable structural dynamic information is required to ensure that the simulated aeroelastic response is representative of the target system. This information may be provided either in the form of experimentally validated modal parameters and mode shapes, or directly through measured structural FRFs.

In this context, the value of spatially dense SLDV measurements is not necessarily that all measured FRFs must be used directly in the final reduced-order aeroelastic model. Ground-based aeroelastic simulation methods often operate on a reduced set of physical coordinates to keep the experimental and numerical formulation manageable. However, high-resolution SLDV-based FRF data can support the selection and validation of these relevant measurement coordinates, reduce interpolation uncertainty, and verify that the reduced structural representation captures the deformation patterns relevant for flutter-critical modal behavior and modal coupling. The presented workflow therefore contributes to the experimental foundation required for future dry wind tunnel or ground-based aeroelastic simulation studies.

7 Conclusions

This paper presented an FE-model-based experimental validation workflow using SLDV for high-resolution modal characterization of a clamped flat-plate structure representative of bending-dominated aerospace components. The FE geometry was used as the spatial reference for the measurement definition, enabling the vibration response to be acquired and visualized directly on the structural model. The resulting FRFs, ODSs and experimentally identified mode shapes provided a dense basis for comparison with numerical modal analysis results.

Experimental modal analysis identified seven modes in the investigated frequency range from 5 Hz to 500 Hz. Comparison with the numerical modal analysis showed good agreement in terms of natural frequencies, with relative deviations below 1.6 % for the correlated mode pairs. The MAC-based correlation and qualitative mode-shape comparison confirmed that the numerical model reproduces the main deformation characteristics of the measured structure. Remaining discrepancies, particularly for modes with more complex spatial behavior or possible in-plane components, highlight the importance of boundary-condition modelling, mesh consistency and measurement direction when interpreting experimental-numerical correlation results.

The study demonstrates that FE-model-based SLDV measurements can reduce the practical gap between high-resolution experimental modal analysis and numerical FE model validation. By linking scan definition, spatial calibration, non-contact vibration measurement, modal analysis and standardized data export, the workflow provides a consistent basis for model updating and further structural dynamic investigations.

Future work should extend the approach to more complex aerospace structures and to measurement configurations capable of capturing three-dimensional vibration response. 3D SLDV systems could improve the correlation of modes with significant in-plane components and enable full-field measurements on larger or geometrically more complex components.

The long-term objective is to support dry wind tunnel testing concepts for aerospace structures. In such a framework, validated structural models would be combined with controlled multi-shaker excitation to reproduce flutter-relevant dynamic loading conditions without requiring a conventional aerodynamic wind tunnel test. Laser Doppler vibrometry systems could also contribute to this future workflow as non-contact sensors for monitoring the structural response during controlled excitation. Multi-fiber or multi-head configurations, combined with digitally synchronized data acquisition such as PTP-based synchronization, may provide a path toward real-time or near-real-time response measurement for closed-loop excitation control. The suitability of such an approach will require further investigation, particularly with respect to signal-processing latency, synchronization accuracy and the robustness of the control strategy.

Acknowledgements

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